

An Introduction to Meta-analysis

Will G Hopkins

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Sport and Recreation, Auckland University of Technology, Auckland 1020, New Zealand. [Email](#).

Reviewer: George A Kelley, Meta-Analytic Research Group, Community Medicine, West Virginia University, Morgantown, WV 26506-9190.

A meta-analysis is a systematic quantitative review of original research studies of some phenomenon, such as the effect of a specific treatment on some aspect of health or behavior. The meta-analyst expresses the magnitudes of effects from all relevant studies in the same units: percent units are best for most effect representing differences or changes in means; risk, odds and hazard ratios are appropriate for proportions; and ratios are appropriate for counts. The meta-analyst then uses an appropriate weighting factor (the inverse of each effect's error variance) to combine the magnitudes into a mean value and its uncertainty (confidence limits). In a traditional meta-analysis, the true effects are assumed to be homogeneous (have the same value) in the analyzed studies, and some "outlier" studies may be eliminated to satisfy this assumption. In the more recent and realistic random-effect or mixed-model meta-analysis, true values of all effects are assumed to be heterogeneous (different), and the analysis provides an estimate of the heterogeneity as a standard deviation representing unexplained typical true variation in the effect between studies. Inclusion of study and mean subject characteristics in the analysis as covariates may reduce heterogeneity and provide further useful information about the magnitude of the effect in different locations and with different subjects. Published effects are usually larger than their true values, owing to the misuse of statistical significance as a criterion for publication. A funnel plot or plot of standardized residuals can reveal such publication bias, and deletion of studies with larger standard errors reduces the bias.

KEYWORDS: Cochrane Collaboration, funnel plot, meta-regression, mixed model, quantitative analysis, random effect, research, systematic review.

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Update Oct 2013. Substantial make-over to take into account the unified approach to ratios of risks or proportions, odds, hazards and counts in the article on [linear models and effect magnitudes](#). A novel approach of including separate effects for each group from controlled trials or other studies with control, reference or other comparison groups is also described.

Update Aug 2007. Minor improvements to slideshow. See also a more [succinct version](#) of the slideshow prepared for but not presented at the 2007 ACSM meeting, as explained in the [conference report](#).

The basis for this article is an updated version of a slideshow accompanying a talk on meta-analysis I presented this year locally and at the University of Bath. The article should meet a need for a straightforward and up-to-date account of meta-analysis suitable for research students and staff in the sport sciences and other biomedical disciplines.

My experience with meta-analysis is limited—one analysis published and three others completed recently—but most of my assertions appear to be consistent with those in the ultimate source of meta-analytic wisdom, the [handbook](#) of the [Cochrane Collaboration](#) (cochrane.org) I depart from the handbook with my emphasis or novel material on

individual responses, standardized differences in means, log transformation, measures of physical performance, and correlations. You will need to refer to the Cochrane handbook for information on topics I don't cover, including survival analysis, intention-to-treat analysis, and meta-analysis of single-subject studies (cases or individual patient data).

The [reprint pdf](#) version of this article contains printer-friendly images of the PowerPoint [slideshow](#) and references to relevant publications.

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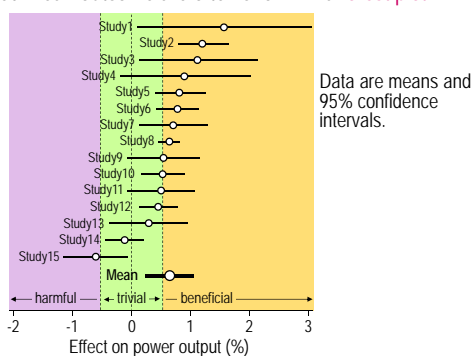
Victoria University, Melbourne, Australia

- What is a Meta-Analysis?
 - Definition, weighted average, heterogeneity, mixed-model meta-regression
- Limitations to Meta-Analysis
 - Individual differences or responses, publication bias
- How to Do a Meta-Analysis
 - Generic measures, finding effects, study characteristics, study quality, weighting factor, model, publication bias
- Summary and References

What is a Meta-Analysis?

- A **systematic review of literature** to address this question: **on the basis of the research to date, how big is a given effect, such as...**
 - the effect of endurance training on resting blood pressure;
 - the effect of bracing on ankle injury;
 - the effect of creatine supplementation on sprint performance;
 - the relationship between obesity and habitual physical activity.
- It is similar to a simple cross-sectional study, in which the **subjects are individual studies** rather than individual people.
 - But the stats are a bit harder.
- A review of literature is a meta-analytic review only if it includes **quantitative estimation** of the magnitude of the effect and its uncertainty (confidence limits).

- The main outcome is the **overall magnitude** of the effect.
- The effect in each study (the **study estimates**) and the meta-analyzed mean outcome are often shown in a **forest plot**:



- The main outcome is **not a simple average** of the study estimates.
 - Meta-analysis uses the **standard error** of each study estimate to give **more weight** to studies with **more precise estimates**.
 - The standard error is the **expected variation** in the study estimate if the study was repeated again and again.
- The weighting factor is $1/(\text{standard error})^2$.
 - Other things being equal, use of this factor is equivalent to weighting the effect in each study by the study's **sample size**.
 - So, for example, a meta-analysis of 3 studies of 10, 20 and 30 subjects each amounts to a single study of 60 subjects.
 - For **controlled trials**, this factor also takes into account differences in **standard error of measurement** between studies.

- You can and should allow for **real differences** or **heterogeneity** in the magnitude of the effect between studies.
 - In early (fixed-effects only) meta-analysis, you did so by testing for heterogeneity using the **Q or chi-squared statistic**.
 - The test has low power, so you used $p < 0.10$ rather than $p < 0.05$.
 - If $p < 0.10$, you excluded **"outlier"** studies and re-tested, until $p > 0.10$.
 - When $p > 0.10$, you declared the effect **homogeneous**.
 - That is, you assumed the differences in the effect between studies were due only to **sampling variation**.
 - Which made it **easy to analyze** the weighted mean effect.
 - But the approach was **unrealistic, limited**, and suffered from the problem of **whether statistical non-significance means negligible**.
- In **random-effect** meta-analysis, you accept there are always **real differences between all studies** in the magnitude of the effect.
 - The "random effect" is the **standard deviation** representing the **variation in the true magnitude** from study to study.

- You get an **estimate of this SD** and its precision.
 - It is sometimes known as **tau** and is provided as **tau²**.
- The mean effect \pm this SD is **what folks can expect typically** in another study or if they try to make use of the effect.
- Instead of this SD, most researchers provide the **Q** and the related **I² statistic**, representing percent of variation due to real differences.
 - **Ignore** any conclusions based on this **uninterpretable** statistic.
- **Mixed-model** meta-analysis or **meta-regression** is best of all.
 - You include study characteristics as **fixed effects**.
 - The study characteristics will partly **account for differences** in the magnitude of the effect between studies.
 - Example: differences between studies of athletes and non-athletes.
 - The random effect now represents **residual variation** in the effect between studies (i.e., not explained by the study characteristics).
 - Include an **extra random effect** to account for repeated measurement (multiple estimates) within studies.
 - **Custom software** or an **advanced package** (e.g., SAS) is required.

Limitations to Meta-Analysis

- It's focused on **mean effects** and **differences between studies**.
 - But what really matters is effects on **individuals**.
 - So we should also quantify **individual differences or responses**.
 - These can be expressed as **standard deviations**, but researchers usually don't provide enough info to allow their meta-analysis.
 - Inclusion of **mean subject characteristics** (e.g., age, gender, genotype) as predictors in the meta-analytic model only partly addresses this problem.
 - It would be better if researchers made available all data for all subjects, to allow **individual patient-data meta-analysis**.
- A meta-analysis reflects only **published effects**.
 - But **statistically significant effects** are more likely to get **published**.
 - Hence published effects are **biased high**.
 - **Funnel** or related plots can be used to assess and reduce publication bias.

How to Do a Meta-Analysis: Opt for a Generic Measure

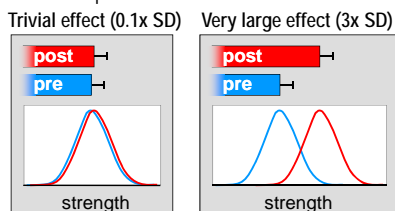
- You can combine effects from different studies only when they are expressed in the **same units**.
- In most meta-analyses, the effects are converted to a generic **dimensionless measure**.

Main measures:

 - standardized difference or change in the mean (Cohen's *d*);
 - Other forms are similar or less useful (Hedges' *g*, Glass's *d*)
 - percent or factor difference or change in the mean;
 - correlation coefficient and slope;
 - risk, odds, hazard and count ratios.

Standardized Difference or Change in the Mean

- Express the difference or change in the mean as a fraction of the **between-subject standard deviation** ($\Delta\text{mean}/\text{SD}$).
- Also known as **Cohen's *d*** (*d* stands for *difference*).
- This example of the effect of a treatment on strength shows why the SD is important:



- The $\Delta\text{mean}/\text{SD}$ are **biased high for small sample sizes** and need correcting before including in the meta-analysis.

- A problem with standardization:
 - Study samples are often drawn from **populations with different SDs**, so some differences in effect size between studies will be **due to the differences in SDs**.
 - Such differences are **irrelevant** and tend to **mask more interesting differences**.
- The solution:
 - Meta-analyze a better generic measure reflecting the **biological effect**, usually **percent** or factor differences or changes.
 - Rarely, the **raw measure** is best; for example, joint angles representing flexibility.
 - **Combine the between-subject SDs** from the studies selectively and appropriately, to get one or more population SDs.
 - Express the overall effect from the meta-analysis as a standardized effect using this/these SDs.
 - This approach also effectively **eliminates the correction for sample-size bias** with standardized effects.

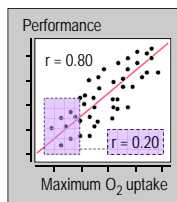
Percent or Factor Difference or Change in the Mean

- The magnitude of many effects can be expressed as a **percent or multiplicative factor** that tends to have the same value for every individual.
 - Example: effect of a treatment on performance is +2%, or a factor of 1.02, regardless of the raw value of the performance.
- For such effects, **percent difference or change** can be the most appropriate generic measure in a meta-analysis.
- If all the studies have small percent effects (<10%), use percent effects **directly** in the meta-analysis.
- Otherwise express the effects (and their standard errors) as **factors** and **log-transform them** before meta-analysis.
 - **Back-transform** the outcomes into percents or factors.
 - Or calculate **standardized differences or changes** in the mean using the log transformed effects and logs of factor SD.

- Measures of **athletic performance** need special care.
 - The best generic measure is **percent change**.
 - But a given percent change in an athlete's ability to output power can result in **different percent changes** in performance in **different exercise modalities**.
 - Example: a 1% change in endurance power output produces the following changes...
 - 1% in running time-trial speed or time;
 - -0.4% in road-cycling time-trial time;
 - 0.3% in rowing-ergometer time-trial time;
 - -15% in time to exhaustion in a constant-power test.
 - So convert all published effects to changes in **power output**.
 - A difficult and time-consuming task; you have been warned!
 - See recent meta-analyses by my students and colleagues.
- For **team-sport fitness tests**, convert percent changes back into standardized mean changes after meta-analysis.

Correlation Coefficient and Slope

- These measures of association between **two numeric variables** are seldom meta-analyzed.
- Studies with **small between-subject SD** have **small correlations**, so correlation suffers from a similar SD problem as standardized effects.
- Solution: meta-analyze the **slope**.
 - The slope is **biased low** (degraded) only by random error in the predictor.
 - Adjust for this bias by dividing the slope by the short-term reliability intraclass correlation coefficient.
 - Express the meta-analyzed slope as either...
 - a **correlation using SD for an appropriate population**, or
 - the effect of **two SD of the predictor** in that population.



Risk, Odds, Hazard and Count Ratios

- When the dependent variable is a **proportion** or **count** of something, effects should be expressed as **ratios**.
- **Risk ratio**, relative risk, proportion ratio...
 - Example: if proportions of inactive and active adults who get heart disease after 20 years are 25% and 10%, risk ratio = $25/10 = 2.5$.
- If proportions are **time-independent classifications**, convert all effects to **odds ratios** for meta-analysis.
 - Convert meta-analyzed odds ratios back into **proportions and proportion ratios** by choosing a sensible proportion for the reference group.
- **Hazard ratio** is the risk ratio for new occurrences in the next brief instant of time (the "right-now" risk ratio).
 - If **proportions change with time**, the proportion and odds ratios also change, but the hazard ratio usually doesn't.

- So, to meta-analyze studies with different time periods, **convert any proportion and odds ratios to hazard ratios**.
- Odds ratios from **time-dependent case-control studies** are already hazard ratios, if controls were sampled as the cases came in (incidence-density sampling).
- If proportions in the two groups in all studies are **low (<10%)**, all proportion, odds and hazard ratios are effectively equal and need not be interconverted.
- Express effects on counts as **count ratios**.
- Express **standard errors** of ratio effects as \times/\div **factor errors**, then log transform the ratios and errors for meta-analysis.
 - Some software does the log transformation and works out the standard errors for you.

How to Do a Meta-Analysis: Find and Record Effects

- Do a **search** of the literature for studies of a specific effect.
 - If the effect has been **meta-analyzed already**...
 - You can do another, if the analysis was done badly or if there have been many new studies since the previous meta.
 - Otherwise find another effect to meta-analyze.
 - As you assemble the published papers, **broaden or narrow the focus** of your review to make it manageable and relevant via...
 - design (e.g., only randomized controlled trials), population (e.g., only competitive athletes), treatment (e.g., only acute effects)...
 - **Document** your searches, inclusions and exclusions.
- Record each **effect magnitude** and inferential information (sample size, p value, confidence limits, SD of change scores).
 - Convert effects into values on a **single scale** of magnitude.
 - In studies with a control or other reference group, record the effect and inferential information **in each group** to enhance the analysis.

How to Do a Meta-Analysis: Get Study Characteristics

- Record **study characteristics** that might account for differences in the effect magnitude between studies.
- Include the study characteristics as **covariates** in the meta-analysis. Examples:
 - duration or dose of treatment;
 - method of measurement of dependent variable;
 - quality score;
 - gender and mean characteristics of subjects (age, status...).
 - Record **separate outcomes** for females and males from the same study, if possible.
 - Otherwise analyze gender as a **proportion** of one gender; for example, in a study of 3 males and 7 females, "Maleness" = 0.3.
 - Use this approach for all problematic **dichotomous characteristics** (sedentary vs active, non-athletes vs athletes, etc.).

How to Do a Meta-Analysis: Assess Study Quality?

- Most meta-analysts score the **quality** of a study.
 - Examples (scored yes=1, no=0):
 - Published in a peer-reviewed journal?
 - Experienced researchers?
 - Research funded by impartial agency?
 - Study performed by impartial researchers?
 - Subjects selected randomly from a population?
 - Subjects assigned randomly to treatments?
 - High proportion of subjects entered and/or finished the study?
 - Subjects blind to treatment?
 - Data gatherers blind to treatment?
 - Analysis performed blind?
 - Use the score to **exclude some studies**, and/or...
 - **Include** as a **covariate** in the meta-analysis, but...
 - Some statisticians advise **caution** when using quality.

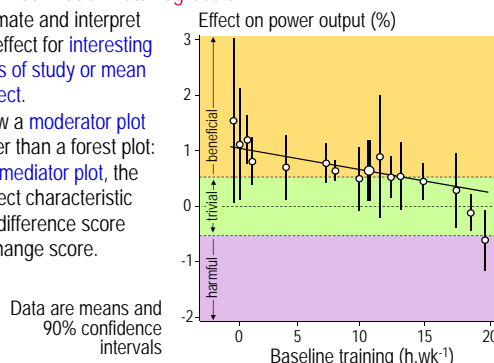
How to Do a Meta-Analysis: Get the Weighting Factor

- Calculate the **standard error** for each effect via one or more of...
 - the **confidence interval** or limits
 - the **test statistic** (t , χ^2 , F)
 - F ratios with numerator degrees of freedom >1 can't be used.
 - the **p value**
 - If the **exact** p value is not given and you can't calculate the standard error from the data, try contacting the authors for it.
 - Otherwise, if " $p < 0.05$ ", analyze as $p = 0.05$.
 - If " $p > 0.05$ " with no other info, deal with the study qualitatively.
- **SD of change scores** (for controlled trials)
 - For studies lacking sufficient information to calculate standard errors, calculate the **typical error** (standard error of measurement) in every other study and impute typical errors (and standard errors via SD of change scores) from these. The spreadsheet for sample-size estimation at SportsScience calculates the typical errors.
- the **data**: done for you by the software, depending on the effect.

How to Do a Meta-Analysis: Develop the Model

- Do a **mixed-model meta-regression**.

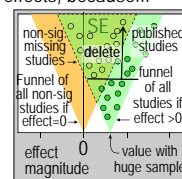
- Estimate and interpret the effect for **interesting types of study or mean subject**.
- Show a **moderator plot** rather than a forest plot:
- In a **mediator plot**, the subject characteristic is a difference score or change score.



- For any linear covariate, estimate and interpret the effect of **2x the average of between-subject SD** from appropriate studies.
- **Double the SD** representing the **between-study random effect** to interpret its magnitude as the unexplained **typical differences** in the magnitude of the effect **between settings**.
 - The random effect provided as τ^2 may be in log-transformed units and may need back-transformation to a factor SD.
 - Apply the p value for τ^2 directly to the SD to get confidence limits.
- For effects where there are **control or reference groups**...
 - include **each group effect** separately, if possible;
 - include a **within-study random effect** to account for the resulting **repeated measurement**;
 - include fixed effects to estimate **uncontrolled effects** and **effects relative to control**, best-practice or other reference groups.
- Inspect between-subject SD between and within studies for evidence of **individual differences or responses**.

How to Do a Meta-Analysis: Deal with Publication Bias

- Some meta-analysts present the effect magnitude of all the studies as a **funnel plot**, to address the issue of **publication bias**.
 - Published effects tend to be **larger** than true effects, because...
 - effects that are larger simply because of **sampling variation** have smaller p values,
 - and $p < 0.05$ is more likely to be **published**.
 - A plot of **standard error vs effect magnitude** should have a triangular or **funnel** shape.
 - If some non-significant studies weren't published, the plot will be **asymmetrical**.
 - The **missing studies** are generally smaller (therefore **larger SE**).
 - **Effect heterogeneity** also disrupts the funnel shape.
 - So plot **standardized residuals (random-effect solution) vs standard error** (not shown) to spot publication bias and also outlier studies.
- **Delete studies** with larger SE to give a **symmetrical plot**.



Summary

- Meta-analysis is a **statistical literature review** of magnitude of an effect.
- Meta-analysis uses the magnitude of the effect and its precision from each study to produce a **weighted mean**.
- **Traditional** meta-analysis is based unrealistically on using a test for heterogeneity to **exclude outlier studies**.
- **Random-effect (mixed-model)** meta-analysis **estimates heterogeneity** and allows estimation of the **effect of study and subject characteristics** on the effect.
- For the analysis, the effects have to be converted into the same units, usually **percent** or other **dimensionless generic measure**.
- It's possible to account for **publication bias** and identify outlier studies using a **funnel plot** or **residuals plot**.

References

- Best: read recent meta-analyses co-authored by me.
 - A good source of meta-analytic wisdom is the **Cochrane Collaboration**, an international non-profit academic group specializing in meta-analyses of healthcare interventions.
 - **Website**: <http://www.cochrane.org>
 - **Publication**: The Cochrane Reviewers' Handbook (2004). <http://www.cochrane.org/resources/handbook/index.htm>.
 - But the (free) Cochrane **meta-analysis software** is somewhat limited.
- These references are getting out of date:
- **Simpler reference**: Bergman NG, Parker RA (2002). Meta-analysis: neither quick nor easy. BMC Medical Research Methodology 2, <http://www.biomedcentral.com/1471-2288/2/10>.
 - **Glossary**: Delgado-Rodriguez M (2001). Glossary on meta-analysis. Journal of Epidemiology and Community Health 55, 534-536.
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